

Math 1321: Accelerated Engineering Calculus II

Lab #1, Week #1, Fall 2018

Instructions: Please show all your work and explain your reasoning as appropriate. You are allowed to use any results from lecture as long as they are stated correctly. You are encouraged to work in groups, but your final written solutions must be in your own words.

Name : _____

uID : _____

Group : _____

Due Date : 8/29/18

Question	Points	Score
Sequences and Series	12	
Flourishing Fish	10	
Stimulus Spending	10	
The Snowflake Curve	18	
Total:	50	

1. **Sequences and Series** Consider the sequence

$$\left\{ \sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots \right\}$$

(a) (7 points) Find an expression for a_n , the n^{th} term of the series.

(b) (5 points) What is $\lim_{n \rightarrow \infty} a_n$?

2. **Flourishing Fish** The size of an undisturbed fish population can be modeled by the formula

$$p_{n+1} = \frac{bp_n}{a + p_n}$$

(a) (2 points) If $a > b$, does the sequence converge?

(b) (2 points) If the sequence converges, find $\lim_{n \rightarrow \infty} p_n$. If it diverges, describe how it diverges. What does this mean in terms of the fish population?

(c) (2 points) Now assume that $a < b$. Show that if $p_0 < b - a$ then $\{p_n\}$ is increasing and $0 < p_n < b - a$.

(d) (2 points) Again assuming that $a < b$, show that if $p_0 > b - a$ then the population is decreasing and $p_n > b - a$.

(e) (2 points) When $a < b$, what is $\lim_{n \rightarrow \infty} p_n$? What does this mean in terms of the fish population?

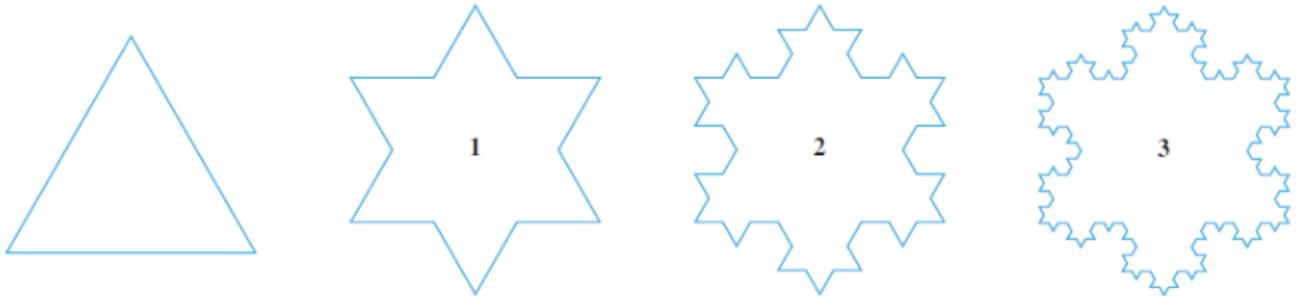
3. **Stimulus Spending** When people receive money, they tend to save some of it while spending the rest. The people who receive that spent money will also spend some of it. The people who receive the twice-spent money will also spend some, and so on. Economists call this chain reaction of spending the multiplier effect. In a hypothetical, isolated community, suppose the local government decides to stimulate spending by distributing D dollars to its population. Assume each recipient of the money saves some proportion, s , and spends the remaining proportion, c , of the money they receive. The values s and c are known as the *marginal propensity to save* and the *marginal propensity to consume* respectively.

(a) (4 points) Find the equation for S_n , the total money spent after n rounds of spending.

(b) (4 points) What is $M = \lim_{n \rightarrow \infty} S_n$?

(c) (2 points) Sketch a graph of the value of M with s on the x -axis and M on the y -axis.

4. **The Snowflake Curve** To construct the snowflake curve, start with an equilateral triangle with sides of length 1. Step 1 in the construction is to divide each side into three equal parts, construct an equilateral triangle on the middle part, and then delete the middle part (see the figure). Step 2 is to repeat step 1 for each side of the resulting polygon. This process is repeated at each succeeding step. The snowflake curve is the curve that results from repeating this process indefinitely.



- (a) (7 points) Let s_n , l_n and p_n represent the number of sides, the length of a side and the total length of the n th approximating curve (the curve obtained after step n of the construction), respectively. Find formulas for s_n , l_n and p_n .

(b) (4 points) Show that $p_n \rightarrow \infty$ as $n \rightarrow \infty$

(c) (7 points) Sum an infinite series to find the area enclosed by the snowflake curve.

(Note: Parts (b) and (c) show that the snowflake curve is infinitely long but encloses only a finite area.)